When a Local Lemma is Best Possible

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Abstract

In 1985, Shearer gave a general theorem characterizing the family L of dependency graphs labeled with probabilities p_v which have the property that for *any* family of events with a dependency graph from L (whose labels are upper bounds on the probabilities of the events), there is a positive probability that none of the events from the family occur.

We show that, unlike the standard Lovasz Local Lemma—which is less powerful than Shearer's condition on *every* nonempty graph—a recently proved 'Lefthanded' version of the Local Lemma is equivalent to Shearer's condition for all chordal graphs. This gives a simple and efficient algorithm to check whether a given labeled chordal graph is in L.