Predecessors and successors in random mappings with exchangeable in-degrees

We consider the structure of the directed random graph, $G_n^{\hat{D}}$, on n labelled vertices with in-degrees given by a sequence, $\hat{D}_1, \hat{D}_2, ..., \hat{D}_n$, of exchangeable non-negative interger-valued random variables with $\sum_{i=1}^{n} \hat{D}_i = n$. The random digraph $G_n^{\hat{D}}$ is the graphical representation of a corresponding random mapping, $T_n^{\hat{D}} : [n] \to [n]$, with exchangeable in-degree sequence $\hat{D}_1, \hat{D}_2, ..., \hat{D}_n$, and, in some sense, it can also be viewed as an analogue of the general independent degree models from random graph theory. In previous work we have shown that the distribution of the number of cyclic points, the number of components, and the size of a typical component can be expressed in terms of expectations of various functions of $\hat{D}_1, \hat{D}_2, ..., \hat{D}_n$. In this talk we develop exact expressions for the distributions of the number of successors and predecessors of an arbitrary set of vertices in $G_n^{\hat{D}}$ in terms of expectations of various functions of $\hat{D}_1, \hat{D}_2, ..., \hat{D}_n$. These results can be applied to obtain corresponding results in two special examples of $T_n^{\hat{D}}$ which correspond to random mappings with preferential and anti-preferential attachment. This is joint work with J. Jaworski (Adam Mickiewicz University), who was supported by the Marie Curie Intra-European Fellowship No. 236845 (RANDOMAPP) within the 7th European Community Framework Programme.