Variations of Rasmussen's Permanent Approximation

Martin Fürer, The Pennsylvania State University

Abstract

Rasmussen's algorithm for the approximation of permanents is an extremely simple unbiased estimator. As Rasmussen has pointed out, its variance can be huge. Fortunately, the basic idea of this algorithm allows a large number of possible improvements. For some obvious improvements, like the elimination of useless entries in the matrix, as well as for sophisticated variations based on scaling, the variance is still exponential in the worst case.

We propose other improvements for which we conjecture polynomial upper bounds on the variance, which would imply a polynomial time approximation scheme. If true, such an approximation scheme would provide a much faster algorithm than the known Markov chain Monte Carlo approximation scheme of Jerrum, Sinclair and Vigoda. Variations of Rasmussen's algorithm easily extend to generalizations not handled by the Markov chain Monte Carlo method, like counting perfect matchings in non-bipartite graphs.

We propose a new method to handle low degree vertices and show an essential ingredient to any polynomial time approximation scheme based on Rasmussen's approach. A recursive algorithm, has to decompose its graph into connected components, and approximate each component separately. Naturally, this also applies when the input graph itself is connected.