The distribution of the number of vertices in the giant component

Béla Bollobás
Cambridge and Memphis

Over fifty years ago, Erdős and Rényi proved the striking result that the structure of the size-random graph $G(n, m)$ undergoes a phase transition as m, the number of edges, grows from less than $n/2$ to more than $n/2$. For the past quarter of a century, this phase transition has been studied in great detail, with a host of detailed results emerging, such as the size of the scaling window, and the behaviour of the giant component outside this window. In particular, Pittel and Wormald proved a deep and difficult theorem about the asymptotic value and limiting distribution of the number of vertices in the giant component above the scaling window of this phase transition. Later, Nachmias and Peres used martingale arguments to study Karp's exploration process, and obtained a simple proof of a weak form of this result.

In this lecture I shall review some of the major theorems concerning the giant component, and then present the simple proof that Riordan and I have found of the full result of Pittel and Wormald.